1. SIMPLIFY THE FOLLOWING EXPRESSIONS. Show all appropriate work!

1. $\sqrt{50k^{10}n^7}$

2. $(9x^4y^{-6})^{\frac{1}{2}}$ (present the result only with positive exponents)

3. $(3\sqrt{k} - 2)(3\sqrt{k} + 2)$

4. $\frac{x^2 - x - 6}{2x^2 + x - 6}$

5. $\frac{x + 3}{x} - \frac{3x + 1}{x - 2}$

6. $\frac{2x^2 - 18}{6x + 20} + \frac{x - 3}{14}$ (division)

$p. 1: 18\ points$
II. SOLVE OR ANSWER QUESTIONS.

7. Find the EXACT solutions: \((z - 5)^2 = 37\)
   (don’t compute the root on a calculator)
   7. ________________________ (2)

8. What number has to be added to \(t^2 - 5t\) to complete the perfect square trinomial? Write the resulting expression as a binomial squared.
   8. Number to add: __________ (2)  
   \((\underline{\underline{\quad}})^2\) (1)

9. \(\frac{4x + 5}{x} - \frac{x}{3} = \frac{10}{3}\)
   9. ____________________________ (5)

10. \(3t - 4 = \sqrt{3t + 16}\)
    10. ____________________________ (5)

11. \(\log_4(4t - 12) + 3 = 5\)
    11. ____________________________ (4)

p. 2: 19 points
12. Solve the system. For full credit, clearly show your algebraic solution and the exact result in fraction form, where appropriate:

\[
\begin{align*}
6x + 2y &= 4 \\
\frac{x}{2} - \frac{y}{6} &= -1
\end{align*}
\]

12. \( x = \) \( \frac{\text{expression}}{\text{expression}} \) (1)

\( y = \) \( \frac{\text{expression}}{\text{expression}} \) (1)

SHOW ALGEBRAIC METHOD (3)

13. For the function \( R(x) = \frac{3x}{2x-1} \):

a. Find \( R(2) \).

\( R(2) = \) \( \frac{\text{expression}}{\text{expression}} \) (1)

b. For what value of \( x \) is \( R(x) = 3 \)?

\( x = \) \( \frac{\text{expression}}{\text{expression}} \) (2)

c. What is the domain of \( R(x) \)?

\( \text{Domain:} \) \( \frac{\text{expression}}{\text{expression}} \) (2)

d. Find \( R(n + 4) \) and simplify

\( R(n + 4) = \) \( \frac{\text{expression}}{\text{expression}} \) (2)

14. If a quadratic equation \( ax^2 + bx + c = 0 \) has the given number of solution, what can you say about the discriminant component \( (D = b^2 - 4ac) \) in the quadratic formula? For each case, circle the correct response:

a. NO REAL SOLUTIONS \( D > 0 \) \( D < 0 \) \( D = 0 \) (2)

b. TWO DIFFERENT REAL SOLUTIONS \( D > 0 \) \( D < 0 \) \( D = 0 \) (2)

c. TWO IDENTICAL REAL SOLUTIONS (only one number) \( D > 0 \) \( D < 0 \) \( D = 0 \) (2)

\( p. \ 3: \ 18 \text{ points} \)
III. MISCELLANEOUS and APPLICATIONS. Show all appropriate work!

15. Find the formula and complete each table, if:

A. \( y \) varies directly with \( x^2 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Formula: (3)

B. \( y \) varies inversely with \( x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>3.5</td>
</tr>
<tr>
<td>20</td>
<td></td>
</tr>
</tbody>
</table>

Formula: (3)

16. In the list below, circle the expression equivalent to \( \log_b \left( \frac{2k}{N^3} \right) \)

\[
\frac{\log_b 2 + \log_b K}{3\log_b N} \quad \log_b (2 + K - 3N) \quad \log_b 2 + \log_b K - 3\log_b N \quad 2\log_b K - N\log_b 3
\]

17. Given the line: \(-8x + 15y = 200\), find the following:

A. The slope of a parallel line

Parallel: ________________ (2)

B. The slope of the perpendicular line

Perpendicular: ________________ (2)

C. The \( y \)-intercept of the given line as the exact fraction in lowest terms.

\( y \)-intercept: ________________ (2)

p. 4: 17 points
18. A veterinarian needs to prepare 23 pounds of a food supplement that is 26% protein. The clinic has a supply of two food supplements, 15% and 35% protein. She will mix some of each to get the right protein content. Let $K$ be the amount of the 15% supplement and let $N$ be the amount of the 35% supplement to be used in the mix. In the provided box, set up the system of equations that describes this situation. Use the given variable names for full credit!

\[ \begin{align*}
15K + 35N & = 23 \\
0.15K + 0.35N & = 0.26(23)
\end{align*} \]

19. The average cost per item depends on the number of items made during a production run. The function below models the average cost $C$ in dollars per item for a new type of industrial electrical switch when $x$ thousand of them are made: $C(x) = 0.705x^2 - 12.69x + 70.8$. Find the following. Where appropriate, round your answer to 2 decimal places.

a. What is the average cost per switch when 3000 switches are made?

b. How many switches should be made for the lowest average cost per switch?

SHOW THE STEPS (SET UP) OF THE CALCULATION FOR FULL CREDIT!

19a. ____________________ (2)

19b. ______________________ (3)

19c. ______________________ (2)

20. The function $P(t) = 280 \times 0.96^t$ models the expected population in a flock of birds $t$ years since the beginning of 2008. Find the following.

a. How many birds are there in the beginning of 2008?

b. What is the expected population at the beginning of 2014?

20a. ____________________ (2)

20b. ____________________ (2)

20c. ______________________ (4)

\[ p. 5: \ \ 19 \ points \]
IV. GRAPHS

21. Plot the graph of the data points from the following table and match your plot with one of the given graphs:

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.5</td>
<td>0.7</td>
</tr>
<tr>
<td>-0.5</td>
<td>0.3</td>
</tr>
<tr>
<td>1.5</td>
<td>-0.4</td>
</tr>
<tr>
<td>2.5</td>
<td>-0.6</td>
</tr>
</tbody>
</table>

Plot (2)                  Graph match: ____________ (2)

22. The two points shown in the coordinate system to the right are on the graph of a quadratic function. One of the points is the vertex of the parabola \( y = ax^2 + bx + c \).

a. If the coefficient \( a > 0 \), what are the coordinates of the vertex?

\[ \text{Vertex: } ( , ) \] (2)

b. Sketch the line of symmetry for this parabola. Use the symmetry and the other given point to determine the coordinates of a third point on the parabola. Sketch a neat graph of the parabola.

\[ \text{Third point: } ( , ) \] (2)

Graph (2)

BONUS: What are the exact coordinates of the \( y \)-intercept?

\[ \text{y-intercept: } ( , ) \] (3)

p. 6: 10 points & 3 bonus points.