PART I: SIMPLIFY AS MUCH AS POSSIBLE: Show appropriate work.

1. \( \left( 16 c^{12} \right)^{\frac{3}{4}} \)
   
   \( \text{1. } \quad \) (2)

2. \( \frac{52 m^{-7} n^{-4}}{26 m^3 n^{-6}} \) \( \text{(Write your answer using only positive exponents)} \)
   
   \( \text{2. } \quad \) (2)

3. \( \sqrt[3]{27x^{10}y^6} \)
   
   \( \text{3. } \quad \) (2)

4. \( \sqrt{12} + 4\sqrt{27} - 6\sqrt{48} \)
   
   \( \text{4. } \quad \) (2)

5. \( \frac{5y - 10}{3y^2 - 3y} + \frac{8y - 16}{y - 1} \)
   
   \( \text{5. } \quad \) (3)

6. \( \frac{2b}{b - 1} + \frac{2}{b} - \frac{2}{b - 1} \)
   
   \( \text{6. } \quad \) (3)

7. Rewrite the equation in exponential form: \( \log_M N = 7 \)
   
   \( \text{7. } \quad \) (2)
PART II: SOLVE AS INDICATED:  

Show appropriate work.

8. Solve for exact value of $d$: $A = \frac{\pi d^2}{2}$  
   (assume all variables represent positive values)

   \[ d = \sqrt{\frac{2A}{\pi}} \]

9. Solve for $x$: $x^2 - 5 = 2x$  
   (write answer in simplified radical form)

   \[ x = \frac{5 \pm \sqrt{25 - 4(1)(-5)}}{2} = \frac{5 \pm \sqrt{35}}{2} \]

10. Solve for $m$: \[
    \frac{2}{m - 3} = \frac{14}{m^2 - 9} - \frac{1}{m + 3}
    \]
    Be sure to check for extraneous solutions.

    \[ m = \frac{5}{3} \]

11. Solve for $y$: \[
    \sqrt{8 - 2y} = y - 4
    \]
    Be sure to check for extraneous solutions.

    \[ y = \frac{9}{2} \]

12. Solve for exact value of $x$: \[
    \log_2(3x + 1) = 4
    \]

    \[ x = \frac{15}{2} \]

13. Solve for exact value of $t$: \[
    4 \cdot 7^t = 196
    \]

    \[ t = 2 \]
14. Solve the system:
\[ \frac{1}{4}x + \frac{1}{2}y = \frac{1}{4} \]
\[ x + 3y = 4 \]

\[ (x, y) = (___, ___) \] (5)
write answer as an ordered pair

PART III: FOLLOW THE INSTRUCTIONS FOR EACH PROBLEM:

15. Find the equation of the line that passes through \((2, -5)\) and is perpendicular to the line
\[ y = \frac{1}{3}x - 2 \]. Write your answer in slope intercept form.

\[ y = \underline{\phantom{0000}} \] (4)

16. Use the information in the graph below to answer the following questions.

a) \[ h(2) = ? \]
a) \[ \underline{\phantom{0000}} \] (1)

b) For what value(s) of \( x \) is \( h(x) = 0 ? \)
b) \[ \underline{\phantom{0000}} \] (2)

c) Give the RANGE of the function.
c) \[ \underline{\phantom{0000}} \] (2)
17. Given \( h(x) = 5x^2 - 3 \)
   a) Find all \( x \) so that \( h(x) = 42 \)

   a) _________________ (3)

   b) Evaluate \( h(c - 1) \) and simplify the result.

   b) _________________ (3)

18. Work this problem algebraically and show your work. You must show all the correct algebraic work for full credit.
   Given the function \( f(x) = 2x^2 - 6x + 4 \), find the vertex, \( x \) and \( y \) intercepts and draw the graph.

   a) Vertex: (___, ___) (2)

   b) \( y \)-intercept: (___, ___) (1)

   c) \( x \)-intercepts: (___, ___) (___, ___) (2)

   d) Graph \( y = f(x) \) on the grid at the left. (2)
19. The volume of gas inside a balloon changes as its temperature changes in such a way that the volume of the gas is linearly related to its temperature. The table below contains the data of a gas that has been heated and cooled where t represents the temperature in degrees Celsius and V represents the volume in cubic centimeters.

<table>
<thead>
<tr>
<th>t (°C.)</th>
<th>-80</th>
<th>-40</th>
<th>0</th>
<th>40</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>V (cc)</td>
<td>150</td>
<td>250</td>
<td>350</td>
<td>550</td>
<td>650</td>
</tr>
</tbody>
</table>

a) Fill in the blank spots in the table. 

b) Using the data from the table, write a linear equation that expresses the volume of the gas, \( V \), in terms of its temperature, \( t \).

\[ V = mt + b \]

\[ m = \frac{650 - 150}{80 - (-80)} = \frac{500}{160} = \frac{25}{8} \]

\[ b = 150 - \left( \frac{25}{8} \right) \cdot (-80) = 150 + 250 = 400 \]

\[ V = \frac{25}{8} t + 400 \]

b) \[ \frac{25}{8} t + 400 \]

(4)

c) Using a complete English sentence, explain what the slope of this line represents in the context of this physical scenario (be sure to include the correct unit of measure).

The slope of \( \frac{25}{8} \) represents the rate of change of the volume of the gas with respect to temperature. It indicates that for every degree Celsius increase in temperature, the volume increases by \( \frac{25}{8} \) cubic centimeters.

(2)

d) Notice that when \( t = -140 \), \( V = 0 \).

Given the model above, which explanation best fits the model? 

(circle one) 

- there’s a hole
- a black hole
- the gas contracted
- in my balloon
- sucked up the gas
- until none remained

(1)

20. Consider the function \( y = -5x^4 - 3x^3 + 3x^2 - 3x + 4 \)

a) Use your graphing calculator to help you sketch a good graph of the function. Size your viewing window with \( x_{\text{min}} = -2 \), \( x_{\text{max}} = 2 \), \( y_{\text{min}} = -5 \), and \( y_{\text{max}} = 10 \). Label the axes in your grid with appropriate units.

b) Use your TRACE, INTERSECT, or ZERO function to find the x-intercepts of the graph that show in your viewing window as accurately as possible.

b) Round answers to the tenth
21. The Purple Haze Lavender Farm is launching an online store and plans to allow their customers to make purchases by credit card. PaySystems, Inc., a credit card processing company, has two fee options available. For each transaction, the first fee option charges the merchant 3.95% of the total purchase plus $1. For each transaction, the second fee option charges the merchant 5.5% of the total purchase plus $0.60.

   a) Let $P$ represent the total purchase. Let $F$ represent the fee that PaySystems charges the merchant for each transaction. Write a linear equation for each of the two fee options.

      Fee with plan 1: ________________________________ (2)

      Fee with plan 2: ________________________________ (2)

   b) What total purchase produces the same fee, regardless of which option is used, and what is that fee? (This is the “point of equilibrium”).

      total purchase: ___________ (2)
      fee: ___________ (1)

22. If the contamination of Puget Sound was suddenly stopped, it has been estimated that the amount of contaminants remaining at time $t$ would be given by the formula $P(t) = 36,500 e^{-0.00382t}$, where $t$ is measured in years and 36,500 ppm (parts per million) is the contamination level at the time further contamination stops.

   a) How much pollution remains after 100 years?

      a) ___________ ___________ (3)

   b) How many years would it take until only 40% of the contaminants remain in Puget Sound?

      b) ___________ ___________ (4)

   c) According to the model above, will the Puget Sound ever be pollution free? YES NO (1)
      circle your answer
23. **Work this problem algebraically** (that is, **without** using the graph, trace, table, or calc functions on your calculator) and show your work. Round any decimal answers to the nearest tenth.

Ima Rocketeer launches a model rocket upwards off the top of her apartment building. The rocket’s distance, \( s(t) \), in feet above the ground after \( t \) seconds is given by the equation

\[
s(t) = -16t^2 + 144t + 160.
\]

a) How long does it take the rocket to reach maximum height?
   \[\text{a) number units (3)}\]

b) What is the maximum height achieved by the rocket?
   \[\text{b) number units (2)}\]

c) How long does it take for the rocket to hit the ground?
   \[\text{c) number units (4)}\]

24. **Write the letter of the graph corresponding to each equation, or write “none” if no graph has the right shape and locations. You should be able to do this without using your calculator!** *(Each response is worth 1 point.)*

a) \( y = \log_3 x \)  a)__________  d) \( y = x^2 + 2 \)  d)__________

b) \( 3x + y = -2 \)  b)__________  e) \( y = 5^{-x} \)  e)__________

c) \( x = -3 \)  c)__________